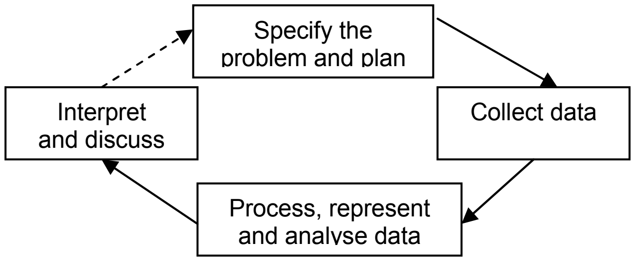
The statistical problem solving cycle

Data are numbers in context and the goal of statistics is to get information from those data, usually through *problem solving*. A procedure or paradigm for statistical problem solving and scientific enquiry is illustrated in the diagram. The dotted line means that, following discussion, the problem may need to be re-formulated and at least one more iteration completed.



Descriptive statistics

Given a sample of n observations $x_1, x_2, \cdots, x_n$ we define the **sample** mean to be

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x_i}{n}$$

and the *corrected* sum of squares by

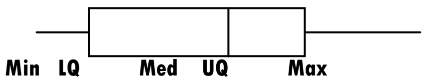
$$S_{xx} = \sum (x_i - \bar{x})^2 \equiv \sum x_i^2 - n\bar{x}^2 \equiv \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}.$$

$\frac{S_{xx}}{n}$ is sometimes called the *mean squared deviation* and an **unbiased estimator** of the population variance σ^2 is $s^2 = \frac{S_{xx}}{(n-1)}$. The **sample standard deviation** is s. In calculating s^2, the divisor (n-1) is called the **degrees of freedom (df).** Note that s is also sometimes written $\hat{\sigma}$.

If the sample data are ordered from smallest to largest then the:

- minimum (Min) is the smallest value;
- lower quartile (LQ) is the ¼(n+1)-th value;
- median (Med) is the middle [or the ½(n+1)-th] value;
- upper quartile (UQ) is the ¾(n+1)-th value;
- maximum (Max) is the largest value.

These five values constitute a **five-number summary** of the data. They can be represented diagrammatically by a *box-and-whisker plot,* commonly called a *boxplot*.



Grouped frequency data

If the data are given in the form of a grouped frequency distribution where we have f_i observations in an interval whose mid-point is x_i then, if $\sum f_i = n$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{n} \text{ and}$$

$$S_{xx} = \sum f_i (x_i - \bar{x})^2 = \sum f_i x_i^2 - \frac{\left(\sum f_i x_i\right)^2}{n}.$$

Events & probabilities

The *intersection* of two events A and B is $A \cap B$. The *union* of A and B is $A \cup B$. A and B are mutually exclusive if they cannot both occur, denoted $A \cap B = \varnothing$, where $\varnothing$ is called the **null event**.

For an event A, $0 \le P(A) \le 1$. For two events A and B $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If A and B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.

Equally likely outcomes

If a complete set of n elementary outcomes are all equally likely to occur, then the probability of each elementary outcome is $1/n$. If an event A consists of m of these n elements, then $P(A) = m/n$.

Independent events

A, B are *independent* if and only if $P(A \cap B) = P(A)P(B)$.

Conditional Probability of A **given** B

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ if } P(B) \neq 0.$$

Bayes' Theorem

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}.$$

Theorem of Total Probability

The k events $B_1, B_2, \ldots, B_k$ form a *partition* of the sample space S if $B_1 \cup B_2 \cup B_3 \ldots \cup B_k = S$ and no two of the B_i 's can occur together. Then $P(A) = \sum_i P(A \mid B_i)P(B_i)$.

In this case Bayes Theorem generalizes to

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{\sum_i P(A \mid B_i)P(B_i)} \quad (i = 1, 2, \ldots, k).$$

If B' is the *complement* of the event B, $P(B') = 1 - P(B)$ and $P(A) = P(A \mid B)\ P(B) + P(A \mid B')P(B')$ is a special case of the theorem of total probability. Also, the complement of the event B is commonly denoted $\bar{B}$.

Guides to Statistical Information 1

Probability & Statistics Facts and Formulae

The Higher Education Academy Maths, Stats & OR Network

Resources to support the learning of mathematics, statistics and OR in higher education.

www.mathstore.ac.uk

The Statistical Education through Problem Solving (STEPS) glossary
www.stats.gla.ac.uk/steps/glossary/

Further reading

Kotz, S. and Johnson, L. (1988) Encyclopedia of Statistical Sciences, Vols 1 – 9. New York: John Wiley and Sons.

Permutations and combinations

The number of ways of selecting *r* objects out of a total of *n*, where the order of selection is important, is the number of permutations: ${}^nP_r = \dfrac{n!}{(n-r)!}$.

The number of ways in which *r* objects can be selected from *n* when the order of selection is not important is the number of combinations: ${}^nC_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$.

${}^nC_n$ must equal 1, so 0!=1 and ${}^nC_0 = 1$; ${}^nC_r = {}^nC_{n-r}$;

${}^nC_0 + {}^nC_1 + \ldots {}^nC_{n-1} + {}^nC_n = 2^n$; ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$.

Random variables

Data arise from observations on variables that are **measured** on different **scales**. A *nominal* scale is used for named categories (eg race, gender) and an *ordinal* scale for data that can be ranked (eg attitudes, position) - no arithmetic operations are valid with either. *Interval* scale measurements can be added and subtracted only (eg temperature), but with *ratio* scale measurements (eg age, weight) multiplication and division can be used meaningfully as well. Generally, random variables are either *discrete* or *continuous*. Note: all data are discrete because the accuracy of measuring is always limited.

A **discrete** random variable *X* can take one of the values $x_1, x_2, \cdots$, the probabilities $p_i = P(X_i = x_i)$ must satisfy $p_i \geq 0$ and $p_1 + p_2 + \ldots = 1$. The pairs (x_i , p_i) form the **probability mass function (pmf)** of *X*.

A **continuous** random variable *X* takes values *x* from a continuous set of possible values. It has a **probability density function (pdf)** f(*x*) that satisfies *f*(*x*) ≥0 and

$\int f(x)dx = 1$, with $P(a < x \leq b) = \int_a^b f(x)dx$.

Expected values

The expected value of a function $H(X)$ of a random variable X is defined as

$$\mathbf{E}\left[H(X)\right] = \begin{cases} \sum H(x_i)P(X = x_i), & X \text{ discrete}, \\ \int H(x)f(x)dx, & X \text{ continuous}. \end{cases}$$

Expectation is linear in that the expectation of a linear combination of functions is the same linear combination of expectations. For example,

E[X^2+ log *X* +1] = **E**[X^2] + **E**[log *X*] + 1,

but **E**[log *X*] ≠ log**E**[*X*] and **E**[1/ *X*] ≠ 1/**E**[*X*].

Variance

The variance of a random variable is defined as

$$Var(X) = \mathbf{E}\left[(X-\mu)^2\right] \equiv \mathbf{E}\left[X^2\right] - \mu^2.$$

Properties:

Var(*X*)$\geq$0 and is equal to 0 only if *X* is a constant.

Var(*a X* + *b*) = a^2*Var*(*X*), where *a* and *b* are constants.

Moment generating functions

The moment generating function (**mgf**) of a random variable is defined as $M_X(t)$= **E**[exp(*t X*)] if this exists.

E[X^k] can be evaluated as the:

(i) coefficient of $t^r/(r!)$ in the power expansion of $M_X(t)$;

(ii) *r*-th derivative of $M_X(t)$ evaluated at *t* = 0.

Measures of location

The **mean** or **expectation** of the random variable *X* is **E**[*X*] the long-run average of realisations of *X* .

The **mode** is where the **pmf** or **pdf** achieves a maximum (if it does so).

For a random variable, *X*, the **median** is such that P(*X* ≤ **median**) = ½, so that 50% of values of *X* occur above and 50% below the **median**.

Percentiles

x_p is the **100-*p*-th percentile** of a random variable *X* if *P*($X \leq x_p$) = p. For example, the 5th percentile, $x_{0.05}$, has 5% of the values smaller than or equal to it. The **median** is the 50-th percentile, the **lower quartile** is the 25th percentile, the **upper quartile** is the 75th percentile.

Measures of dispersion

The **inter-quartile range** is defined to be the difference between the upper and lower quartiles, **UQ** - **LQ**.

The **standard deviation** is defined as the square root of the variance, $\sigma = \sqrt{Var(X)}$, and is in the same units as the random variable *X*.

Cumulative Distribution Function

This is defined as a function of any real value *t* by

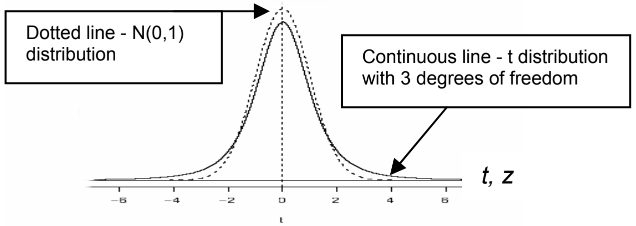
$$F(t) = P(X \leq t).$$

X is a continuous random variable if *F* is a continuous function of *t*; if *X* is discrete, then *F* is a step function.

The Central Limit Theorem

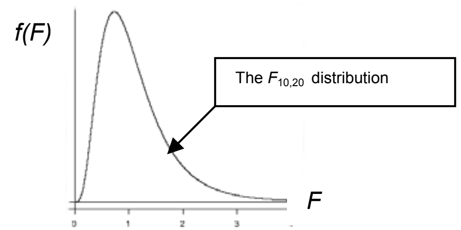
If a random sample of size *n* is taken from *any* distribution with mean μ and variance σ^2, the sampling distribution of the mean will be *approximately* ~N($\mu,\sigma^2/n$), where ‘ ~ ’ means ‘is distributed as’. The larger *n* is, the better the approximation.

The standard normal and Student’s t distributions



If a random variable $X \sim N(\mu,\sigma^2)$, $z = (X-\mu)/\sigma \sim N(0,1)$, the *standard normal distribution*. The *t* distribution with (*n*-1) degrees of freedom is used in place of *z* for small samples size *n* from normal populations when σ^2 is unknown. As *n* increases the distribution of *t* converges to *N*(0,1). These distributions are used, for example, for inference about means, differences between means and in regression.

Fisher’s F distribution



If $X_1 \sim \chi^2_{\nu_1}$ and $X_2 \sim \chi^2_{\nu_2}$ are independent random variables then $\dfrac{X_1/\nu_1}{X_2/\nu_2} \sim F_{\nu_1,\nu_2}$, the *F* distribution with (ν_1, ν_2) degrees of freedom. This distribution is used, for example, for inference about the ratio of two variances, in Analysis of Variance (ANOVA) and in simple and multiple linear regression.

Statistics & Sampling Distributions

Population and samples

A (statistical) **population** is the complete set of all possible measurements or values, corresponding to the entire collection of units, for which inferences are to be made from taking a **sample** - the set of measurements or values that are actually collected from a population.

Simple random sample: every item in the population is equally likely to be in the sample, independently of which other members of the population are chosen.
Parameter: a quantity that describes an aspect of a population, eg. the population mean, μ or variance, σ^2.
Statistic: a quantity calculated from the sample, eg. the sample mean $\bar{x}$ or variance s^2.

Sampling distributions: The value of a statistic will in general vary from sample to sample, in which case it will have its own probability distribution, called its **sampling distribution**. A statistic used to estimate the value of a *parameter* θ in a distribution is called an **estimator** (the random variable) or an **estimate** (the value). If $\hat{\theta}$ is an estimator of θ, the mean of its sampling distribution, **E**[$\hat{\theta}$], is called the *sampling mean* and the variance, *Var*($\hat{\theta}$), is called the *sampling variance*. $\sqrt{Var(\hat{\theta})}$ is called the *standard error* of $\hat{\theta}$. If **E**[$\hat{\theta}$] = θ, then $\hat{\theta}$ is an unbiased estimator of θ. eg, $\bar{X}$ is an unbiased estimator for μ and has sampling variance $\frac{\sigma^2}{n}$ where Var(X_i) = σ^2, ($i = 1,\ldots,n$).

Corrected sum of squares

$$S_{xx} = \sum (x_i - \bar{x})^2 \equiv \sum x_i^2 - n\bar{x}^2 \equiv \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}$$

has **expectation** $(n-1)\sigma^2$ so that dividing S_{xx} by (n–1) will give an unbiased estimator of σ^2, denoted s^2.

Normal and Chi-squared distributions

If $X_1, X_2, \cdots, X_n$ are independently and identically ~N(μ,σ^2), then $\sum\left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi_n^2$ a Chi-squared distribution with n **degrees of freedom**.

Also $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ independently of $\frac{S_{XX}}{\sigma^2} \sim \chi^2_{(n-1)}$.

Simple Linear Regression

To fit the straight line $y = \alpha + \beta x$ to data (x_i, y_i), $i = 1,\ldots,n$, by the method of **least squares** the estimates of slope, β, and intercept α, are given by:

$$b = \frac{\sum x_i y_i - \frac{1}{n}\left(\sum x_i \sum y_i\right)}{\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2} = \frac{S_{xy}}{S_{xx}},$$

$$a = \bar{y} - b\bar{x}.$$

If we assume that the x_i are known and that the y_i have normal distributions with means $\alpha + \beta x_i$, and constant variance σ^2, written as $y_i \sim N(\alpha + \beta x_i, \sigma^2)$, then if x_0 is a fixed value

$$b \sim N\left(\beta, \frac{\sigma^2}{S_{xx}}\right),$$

$$a \sim N\left(\alpha, \sigma^2\left\{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right\}\right),$$

$$a + bx_0 \sim N\left(\alpha + \beta x_0, \sigma^2\left\{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right\}\right).$$

A common alternative is to use $\hat{\alpha}$ for a and $\hat{\beta}$ for b.

Correlation

Given observations (x_i, y_i), $i = 1,\ldots,n$ on two random variables X and Y, the **Pearson (product moment)** correlation between them is given by:

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{1}{n}\left(\sum x_i \sum y_i\right)}{\sqrt{\left\{\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2\right\}\left\{\sum y_i^2 - \frac{1}{n}\left(\sum y_i\right)^2\right\}}}.$$

We use r to estimate the correlation, ρ, between X and Y. For large n, r is approximately ~ $N\left(\rho, \frac{1}{n-2}\right)$. The (**Spearman**) Rank Correlation Coefficient is given by

$$r_S = 1 - \frac{6\sum d_i^2}{n\left(n^2 - 1\right)}.$$

where d_i is the difference between the *ranks* of (x_i, y_i), $i = 1,\ldots,n$. If ranks are tied, see further reading.

Time Series

A time series Y_t (t=1,2, …, n) is a set of n observations recorded through time t, (eg. days, weeks, months). The arithmetic mean of blocks of k successive values $(Y_1+Y_2+\ldots+Y_k)/k$, $(Y_2+Y_3+\ldots+Y_{k+1})/k$,… is a **simple moving average** of order k, itself a time series which is *smoother* than Y_t and can be used to track, or estimate, the underlying level, μ_t, of Y_t. If 0<α<1 an **exponentially weighted moving average** (EWMA) at time t uses a discounted weighted average of current and past data to estimate μ_t with $\hat{\mu}_t = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + \ldots$.
This is equivalent to the *recurrence relation* $\hat{\mu}_t = \alpha Y_t + (1-\alpha)\hat{\mu}_{t-1}$. Moving averages are often plotted on the same graph as Y_t. If Y_t additionally contains trend, R_t, the rate of change of data per unit time, and $\mu_t = \mu_{t-1} + R_{t-1}$, then the recurrence relation is $\hat{\mu}_t = \alpha Y_t + (1-\alpha)(\hat{\mu}_{t-1} + \hat{R}_{t-1})$. If 0< β <1 the *trend smoothing equation is* $\hat{R}_t = \beta(\hat{\mu}_t - \hat{\mu}_{t-1}) + (1-\beta)\hat{R}_{t-1}$, known as *Holt's Linear Exponential Smoothing*. If Y_t also contain *seasonality*, S_t, a smoothing constant γ (0< γ <1) is used in a *seasonal smoothing equation*,
$\hat{S}_t = \gamma Y_t / \hat{\mu}_t + (1-\gamma)\hat{S}_{t-k}$, assuming the periodicity is k, with *multiplicative* seasonality. For monthly data k=12.
Forecasting from time *n* (now) to time *n+h* (*h*=1,2,..)
Level only, $\hat{Y}_{n+h} = \hat{\mu}_n$, the latest EWMA.
Level and constant trend, $\hat{Y}_{n+h}$ = a + b(n+h), the simple linear regression trend line of Y_t against t.
Level and changing trend, $\hat{Y}_{n+h} = \hat{\mu}_n + h\hat{R}_n$.
Level, changing trend and seasonality $\hat{Y}_{n+h} = \hat{\mu}_n + h\hat{R}_n$,
where $\hat{\mu}_n = \alpha Y_n / \hat{S}_{n-12} + (1-\alpha)(\hat{\mu}_{n-1} + \hat{R}_{n-1})$.

A **hypothesis test** involves testing a claim, or **null hypothesis H_0,** about a parameter against an alternative, **H_1**. A decision to **reject H_0** or **not reject H_0** uses sample evidence to *calculate* a **test statistic** which is judged against a **critical value**. **H_0** is maintained unless it is made untenable by sample evidence. Rejecting H_0 when we should not is a **Type I error**. The probability (we are prepared to accept) of making a Type I error is called the **significance level** α and yields the critical value. The *smallest* α at which we can just reject H_0 is the **p-value** of the test, the **tail area** outside the test statistic. Not rejecting H_0 when we should is a **Type II error,** with probability β**.** The **power** of a hypothesis test is 1- β. An **interval estimate** for a parameter is a *calculated* range within which it is deemed likely to fall. Given α, the set of intervals from infinitely repeated random samples of size *n* will contain the parameter (100-α)% of the time: each interval is a (100-α)% **confidence interval**.

One sample hypothesis tests

1. *For $X \sim N(\mu,\sigma^2)$, σ^2 known; random sample evidence $\bar{x}$ and n.*
Null hypothesis, H_0: $\mu=\mu_0$; 2-sided alternative H_1: $\mu\neq\mu_0$.
Test statistic $z_{calc} = (\bar{x} - \mu_0)/(\sigma/\sqrt{n}) \sim N(0,1)$.
Reject H_o (at the α level) if $|z_{calc}| \geq z_{\alpha/2}$, the critical value of *z*.

2. *For $X \sim N(\mu,\sigma^2)$, σ^2 unknown; sample evidence $\bar{x}$, s and n.*
Null hypothesis, H_0: $\mu=\mu_0$; 2-sided alternative H_1: $\mu\neq\mu_0$.
Test statistic $t_{calc} = (\bar{x} - \mu_0)/(s/\sqrt{n}) \sim t_{(n-1)}$ the *t* distribution with (*n*-1) df. For *n*>30 and if *X* has *any* distribution, $t \sim N(0,1)$.
Reject H_o if $|t_{calc}| \geq t_{\alpha/2}$ the critical value of *t* with (*n*-1) df.

3. *For $X \sim N(\mu,\sigma^2)$, σ^2 unknown; sample evidence s and n.*
Null hypothesis, $H_0 : \sigma^2=\sigma_0^2$; alternative H_1: $\sigma^2>\sigma_0^2$.
Test statistic $\chi^2_{calc} = (n-1)s^2/\sigma_0^2 \sim \chi^2_{n-1}$.
Reject H_o if $\chi^2_{calc} > \chi^2_{\alpha}$, the critical value of χ^2 with (*n*-1) df.

Two sample hypothesis tests

For $X_1 \sim N(\mu_1,\sigma_1^2)$, $X_2 \sim N(\mu_2,\sigma_2^2)$, σ_1^2, σ_2^2 unknown; random sample evidence $\bar{x}_1$, $\bar{x}_2$, s_1^2, s_2^2, n_1, and n_2.

1. Null hypothesis, H_0: $\mu_1-\mu_2=c$; 2-sided alternative H_1: $\mu_1-\mu_2\neq c$.
Test statistic $t_{calc} = (\bar{x}_1 - \bar{x}_2 - c)/s\sqrt{(1/n_1 + 1/n_2)} \sim t_{(n_1+n_2-2)}$, and
$s^2 = \{(n_1-1)s_1^2+(n_2-1)s_2^2\}/(n_1+n_2-2)$, assuming $\sigma_1^2=\sigma_2^2$.
Reject H_o if $|t_{calc}| \geq t_{\alpha/2}$ the critical value of *t* with (n_1+n_2-2) df.

2. Null hypothesis H_0: $\sigma_1^2=\sigma_2^2$; alternative H_1: $\sigma_1^2>\sigma_2^2$.
Test statistic $F_{calc} = (n_1-1)s_1^2/(n_2-1)s_2^2 \sim F_{n_1-1,n_2-1}$.
Reject H_0 if $F_{calc} > F_{\alpha}$ the critical value of *F* with n_1-1and n_2-1 df.

Confidence interval for a population mean

If *X* has mean μ and variance σ^2, with n>30 an approximate (100-α)% confidence interval for μ is $\bar{x} - t_{\alpha/2}\, s/\sqrt{n}$ to $\bar{x} + t_{\alpha/2}\, s/\sqrt{n}$. If $X \sim N(\mu,\sigma^2)$ the interval is exact for all *n*.

Standard statistical distributions

Name/parameters	Conditions/application	pdf/pmf	Mean	Variance	mgf	Notes
Binomial Bin(*n*,*p*) Positive integer *n* Probability *p*, $0\leq p\leq 1$	*n* independent success/fail trials each with probability *p* of success. *X* = number of successes.	$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$ *x* = 0,1,…, *n*	np	$np(1-p)$	$\{1-p+pe^t\}^n$	$X \sim \text{Bin}(n,p) \Rightarrow n-X \sim \text{Bin}(n,1-p)$
Geometric Geom(*p*) Probability *p*, $0\leq p\leq 1$	Repeated independent success/fail trials each with probability *p* of success. *X* = number of trials up to and including the first success.	$P(X=x)=(1-p)^{x-1}p$ *x* = 1,2,…	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$	Has the "lack of memory" property $P(X > a+b \mid X > b) = P(X > a)$
Poisson Po(λ) λ a positive number	Events occur at random at a constant rate. *X* = number of occurrences in some interval. λ is the expected number of occurrences.	$P(X=x)=e^{-\lambda}\frac{\lambda^x}{x!}$ *x* = 0,1,2,…	λ	λ	$\exp\{\lambda(e^t-1)\}$	Useful as approximation to Bin(*n*,*p*) if *n* is large and *p* is small
Normal $N(\mu,\sigma^2)$ μ, σ both real; σ>0.	A widely used distribution for symmetrically distributed random variables with mean μ and standard deviation σ.	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$ all real *x*	μ	σ^2	$\exp\left\{\mu t+\frac{1}{2}\sigma^2t^2\right\}$	Can approximate Binomial, Poisson, Pascal and Gamma distributions (See Central Limit Theorem)
Exponential Expon(θ)	If events are occurring at rate θ per unit time. *X* = time to first occurrence.	$f(x)=\theta\exp\{-\theta x\}$ *x*>0	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	$\frac{\theta}{\theta-t},\ t<\theta$	Has the "lack of memory" property $P(X > a+b \mid X > b) = P(X > a)$
Negative-binomial or Pascal Pasc(*r*,*p*) Positive integer *r* Probability *p*, $0\leq p\leq 1$	Repeated independent success/fail trials each with probability *p* of success. *X* = number of trials up to and including the *r*-th success.	$P(X=x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}$ *x* = *r*, *r*+1, *r*+2, …	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left\{\frac{pe^t}{1-(1-p)e^t}\right\}^r$	Pasc(1,*p*) $\equiv$ Geom(*p*)
Gamma Ga(α,β) α, β > 0	Generalization of the exponential distribution; if α is an integer it represents the waiting time to the α-th occurrence of a random event where β is the expected number of events.	$f(x)=\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$ *x*>0	$\frac{\alpha}{\beta}$ α >1	$\frac{\alpha}{\beta^2}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha,\ t<\beta$	Ga(1,λ) $\equiv$ Expon(λ) If ν is an integer Ga(ν/2,2) is χ^2_ν, the Chi-squared distribution with ν df